Clase 17 2 Septiembre 2015

Título de la nota 02/09/2015

Proceso Adiabático Irreversible

$$\Delta U = q - w$$

$$q = 0$$

$$\Delta U = -w$$

$$w_{irrev} = p_2(V_2 - V_1)$$

$$\Delta U = n\bar{C}_V \Delta T$$

$$\Delta H = n\bar{C}_p \Delta T$$

} gas perfecto

$\bar{C}_p$ y $\bar{C}_V$ = ctes

sí $\quad \Delta U = -W$

$$n\bar{C}_v \Delta T = -p\Delta V$$

$$n\bar{C}_v\, dT = -p\, dV \qquad p = p_2$$

$$n\bar{C}_v (T_2 - T_1) = -p_2 (V_2 - V_1)$$

$$\cancel{n}\bar{C}_v (T_2 - T_1) = -p_2 \left[\frac{\cancel{n}RT_2}{p_2} - \frac{\cancel{n}RT_1}{p_1} \right]$$ eliminando

$$p_2 \left[\frac{T_1}{p_1} R - \frac{R T_2}{p_2} \right]$$ Factorizando

$$\bar{C}_v(T_2-T_1) = R\left[\frac{p_2 T_1}{p_1} - T_2\right]$$

Agrupando

$$T_2 - T_1 = \frac{R}{\bar{C}_v}\left[\frac{p_2 T_1}{p_1} - T_2\right]$$

$$T_2 = \frac{R}{\bar{C}_v}\left[\frac{p_2 T_1}{p_1} - T_2\right] + T_1$$

$$T_2 = \frac{\bar{C}_p - \bar{C}_v}{\bar{C}_v}\left[\frac{p_2 T_1}{p_1} - T_2\right] + T_1$$

$$T_2 = \gamma - 1 \left[\frac{p_2 T_1}{p_1} - T_2 \right] + T_1$$

$$T_2 + T_2(\gamma - 1) = \gamma - 1 \left[\frac{p_2}{p_1} T_1 \right] + T_1$$

$$T_2 + T_2\gamma - T_2 = T_1 \left[(\gamma - 1)\frac{p_2}{p_1} + 1 \right]$$

$$T_2\gamma = T_1 \left[(\gamma - 1)\frac{p_2}{p_1} + 1 \right]$$

$$T_2 = \frac{T_1}{\gamma} \left[(\gamma - 1)\frac{p_2}{p_1} + 1 \right]$$

Comprobando

$$\boxed{T_2 =} \quad \boxed{\frac{T_1}{\gamma}\left[(\gamma-1)\frac{P_2}{P_1}+1\right]}$$

en Forma general

$$T_2 = \frac{T_1}{X}\left[(X-1)\frac{P_2}{P_1}+1\right]$$

sí Fuera isotérmico

$X = 1$

sustituyendo

$$T_2 = \frac{T_1}{1}\left[(1-1)\frac{P_2}{P_1}+1\right] = T_1\left[+1\right] = T_2$$

Correcto

Tarea para el Jueves

Expansión Irreversible

$$T_2 = \frac{T_1}{\gamma}\left[(\gamma-1)\frac{P_2}{P_1} + 1\right]$$

antes de resolver obtener p_2 (relación irreversible)

$p_1 = 0.7828$ atm ó 595 mmHg

Obtención de la Relación V vs P

$$\frac{T_2}{T_1} = \frac{1}{\gamma}\left[(\gamma-1)\frac{P_2}{P_1} + 1\right]$$

Sustituyendo

$$\frac{P_2V_2/\cancel{n}R}{P_1V_1/\cancel{n}R} = \frac{1}{\gamma}\left[(\gamma-1)\frac{P_2}{P_1}+1\right]$$

$$P_2V_2 = \frac{P_1V_1}{\gamma}\left[(\gamma-1)\frac{P_2}{P_1}+1\right]$$

$$P_2V_2 = \left[\frac{\cancel{P_1}V_1}{\gamma}(\gamma-1)\frac{P_2}{\cancel{P_1}} + \frac{P_1V_1}{\gamma}\right]$$

eliminando

$$P_2V_2 = \left[\frac{V_1}{\gamma}(\gamma-1)P_2 + \frac{P_1V_1}{\gamma}\right]$$

$$p_2 V_2 \gamma = \left[(\gamma - 1) V_1 P_2 + P_1 V_1 \right]$$

$$p_2 V_2 \gamma = V_1 \left[(\gamma - 1) P_2 + P_1 \right]$$

$$p_2 V_2 = \frac{V_1}{\gamma} \left[(\gamma - 1) P_2 + P_1 \right]$$ dividiendo entre p_2

$$\frac{p_2 V_2}{P_2} = \frac{V_1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_2} + \frac{P_1}{P_2} \right]$$

$$V_2 = \frac{V_1}{\gamma} \left[(\gamma - 1) + \frac{P_1}{P_2} \right]$$ Checar unidades

Tambien se puede obtener como Función de P2

$$p_2 v_2 = \frac{v_1}{\gamma}\left[(\gamma-1)P_2 + P_1\right]$$

$$p_2 v_2 = \frac{v_1}{\gamma}(\gamma-1)p_2 + \frac{v_1 P_1}{\gamma}$$

$$p_2 v_2 - \frac{v_1}{\gamma}(\gamma-1)p_2 = \frac{v_1 P_1}{\gamma}$$

$$p_2\left[v_2 - \frac{v_1}{\gamma}(\gamma-1)\right] = \frac{v_1 P_1}{\gamma}$$

$$P_2 = \boxed{\left\{ \frac{\frac{V_1 P_1}{\gamma}}{\left[V_2 - \frac{V_1}{\gamma}(\gamma - 1) \right]} \right\}}$$

Dividido entre $\dfrac{\gamma}{V_1}$

Cualquier relación de P_2 es válida.

$$\boxed{P_2} = \frac{\frac{V_1}{\gamma} P_1 \frac{\gamma}{V_1}}{\left[V_2 - \frac{V_1}{\gamma}(\gamma - 1) \right] \frac{\gamma}{V_1}} = \boxed{\frac{P_1}{\left[\frac{V_2}{V_1}\gamma - (\gamma - 1) \right]}}$$

Relación T vs V

A partir de

$$T_2 = \frac{T_1}{\gamma}\left[(\gamma-1)\frac{P_2}{P_1}+1\right]$$ Sustituyendo

$$T_2 = \frac{T_1}{\gamma}\left[(\gamma-1)\frac{\cancel{n}RT_2/V_2}{\cancel{n}RT_1/V_1}+1\right]$$ eliminando

$$\frac{T_2}{T_1}\gamma = \left[(\gamma-1)\frac{T_2/V_2}{T_1/V_1} + 1\right]$$

$$\frac{T_2}{T_1}\gamma = \left[(\gamma-1)\frac{T_2 V_1}{T_1 V_2} + 1\right]$$

$$\frac{T_2}{T_1}\gamma = (\gamma-1)\frac{T_2}{T_1}\frac{V_1}{V_2} + 1$$

$$\frac{T_2}{T_1}\gamma - (\gamma-1)\frac{T_2}{T_1}\frac{V_1}{V_2} = 1$$

$$\frac{T_2}{T_1}\left[\gamma - (\gamma-1)\frac{V_1}{V_2}\right] = 1$$

$$\boxed{T_2 = \frac{T_1}{\left[\gamma - (\gamma-1)\frac{V_1}{V_2}\right]}}$$

Forma general
Irreversible

$$T_2 = \frac{T_1}{\left[x - (x-1)\frac{V_1}{V_2}\right]}$$

Tarea: comparar la Exp. Adiab. Rev. contra la Exp. Adiab. Irrev.

Comprobando proceso isobárico Rev e Irrev.

Rev $$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{x-1}{x}}$$

$$\left(\frac{T_2}{T_1}\right)^{x} = \left(\frac{P_2}{P_1}\right)^{x-1}$$ sí $x=0$ isobárico

$$\left(\frac{T_2}{T_1}\right)^{0} = \left(\frac{P_2}{P_1}\right)^{0-1} = 1 = \left(\frac{P_1}{P_2}\right)^{1}$$

$p_2 = p_1$ de forma reversible ✓

Irrev.

$$T_2 = \frac{T_1}{x}\left[(x-1)\frac{P_2}{P_1}+1\right]$$

si $x=0$ isobárico

$$\frac{T_2}{T_1}x = \left[(x-1)\frac{P_2}{P_1}+1\right]$$

No importa como se ataque el proceso isobárico Rev e Irrev. tiene la misma relación

$$\frac{T_2}{T_1}(0) = \left[(0-1)\frac{P_2}{P_1}+1\right]$$

$$0 = -\frac{P_2}{P_1}+1 \quad \therefore \quad -1 = -\frac{P_2}{P_1} = 1 = \frac{P_2}{P_1}$$

$$\boxed{p_2 = p_1} \checkmark$$

Predicción (proceso Adiab. exp.)

$W_{IR} < W_R$ $\qquad$ IR = Irreversible

R = Reversible

$p2_R < p2_{IR}$

$T2_R < T2_{IR}$ se enfría más el reversible

$\Delta S_{IR} > 0$ $\qquad$ $\Delta S_R = 0$ equilibrio

espontáneo

Funciones estado proceso Adiab. Irrev.

gas perfecto

$$\Delta U = n\bar{C_v}(T_2 - T_1)$$

$$\Delta H = n\bar{C_p}(T_2 - T_1)$$

$$q = 0$$

$$w = p_2(v_2 - v_1)$$

como se calcula ΔS_{IR} ?

$\Delta S_{IR} = ?$

aplicando

$$\frac{đq}{T} = \frac{du + đw}{T} = ds$$

$$ds_{IR} = n\bar{C}_v \frac{dT}{T} + \frac{p\,dv}{T} \quad \text{sí} \quad \frac{P}{T} = \frac{nR}{V}$$

$$ds_{IR} = n\bar{C}_v \frac{dT}{T} + nR\frac{dv}{V} \quad \text{Integrando}$$

$$\int_1^2 ds_{IR} = n\bar{C}_V \int_{T_1}^{T_2} \frac{dT}{T} + nR \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\Delta S_{IR} = n\bar{C}_V \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1}$$

$$\Delta S_{IR} > 0$$