

Clase 37 20 octubre 2021

Título de la nota

20/10/2021

Proceso Adiabático Reversible (Deducción de la relación de variables)

$$\text{Si } q = 0 \quad \Delta U = -w$$

$$\Delta U = -w$$

$$dU = -\delta w$$

$$n\bar{C}_v dT = -p dv \quad \text{si es reversible}$$

$$\cancel{n\bar{C}_v dT} = -\cancel{nRT} \frac{dv}{v}$$

$$\bar{C}_v dT = -RT \frac{dv}{v}$$

$$\frac{\bar{C}_v dT}{T} = -R \frac{dv}{v}$$

$$p = \frac{nRT}{v}$$

de acuerdo a la relación de Mayer

$$\bar{C}_p = \bar{C}_v + R$$

$$R = \bar{C}_p - \bar{C}_v$$

y de acuerdo al coeficiente isentrópico

$$\gamma = \bar{C}_p / \bar{C}_v \quad \text{adimensional}$$

$\gamma > 1 \neq 0$ nunca toma valores negativos

Continuando

$$\bar{C}_v \frac{dT}{T} = -R \frac{dv}{v}$$

$$\bar{C}_v \frac{dT}{T} = -(\bar{C}_p - \bar{C}_v) \frac{dv}{v} \quad \therefore$$

$$\frac{dT}{T} = - \left(\frac{\bar{C}_p - \bar{C}_v}{\bar{C}_v} \right) \frac{dv}{v}$$

$$\frac{dT}{T} = -(\gamma-1) \frac{dv}{v}$$

Integrando v_2

$$\int_{T_1}^{T_2} \frac{dT}{T} = -(\gamma-1) \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\ln \frac{T_2}{T_1} = -(\gamma-1) \ln \frac{v_2}{v_1} \quad \ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2} \right)^{\gamma-1}$$

aplicando leyes de Logaritmas

$$e^{\ln\left(\frac{T_2}{T_1}\right)} = e^{\ln\left(\frac{V_1}{V_2}\right)^{\gamma-1}}$$

aplicando función inversa

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} \dots \textcircled{1}$$

de forma general

Sí fuera isotérmico $\gamma=1$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Fórmula correcta

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{1-1} = T_2 = T_1 \left(\frac{V_1}{V_2}\right)^0 = T_2 = T_1 \text{ isotérmico}$$

A partir de ① obtener la relación T y P

$$\frac{T_2}{T_1} = \left(\frac{\cancel{nRT_1/P_1}}{\cancel{nRT_2/P_2}} \right)^{\gamma-1} \quad \text{eliminando} \quad \frac{T_2}{T_1} = \left(\frac{T_1/P_1}{T_2/P_2} \right)^{\gamma-1}$$

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{T_1 P_2}{T_2 P_1} \right)^{\gamma-1} \quad \therefore \quad \frac{T_2}{T_1} = \left(\frac{T_1}{T_2} \right)^{\gamma} \left(\frac{T_1}{T_2} \right)^{-1} \left(\frac{P_2}{P_1} \right)^{\gamma-1}$$

$$\left(\frac{T_2}{T_1} \right)^1 \left(\frac{T_2}{T_1} \right)^{-1} \left(\frac{T_2}{T_1} \right)^{\gamma} = \left(\frac{P_2}{P_1} \right)^{\gamma-1} \quad \left(\frac{T_2}{T_1} \right)^{\gamma}$$

$$\left(\frac{T_2}{T_1}\right)^\gamma = \left(\frac{P_2}{P_1}\right)^{\gamma-1}$$

eliminando

$$\left(\frac{T_2}{T_1}\right)^{\cancel{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma} \therefore T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\gamma-1/\gamma}$$

de forma general

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

si fuera isobárico $\gamma=0$

$$\left(\frac{T_2}{T_1}\right)^\gamma = \left(\frac{P_2}{P_1}\right)^{\gamma-1}$$

$$\left(\frac{T_2}{T_1}\right)^0 = \left(\frac{P_2}{P_1}\right)^{0-1} = 1 = \left(\frac{P_1}{P_2}\right)^1 = P_2 = P_1 \text{ comprobado}$$

Fórmula correcta

$$T = \frac{pV}{nR}$$

De ① obtener la relación P vs V

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\frac{p_2 V_2 / nR}{p_1 V_1 / nR} = \left(\frac{V_1}{V_2} \right)^{\gamma-1} \quad \text{eliminando}$$

$$\left(\frac{p_2 V_2}{p_1 V_1} \right) = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^{\cancel{\gamma-1}} \left(\frac{V_1}{V_2} \right)^{\gamma} \left(\frac{V_1}{V_2} \right)^{\cancel{-1}} \quad \text{eliminando}$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^{\gamma} \quad \therefore \quad p_2 V_2^{\gamma} = p_1 V_1^{\gamma}$$

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^{\gamma} \quad \checkmark$$

Sí fuera isobárico $x=0$

de forma general

$$p_2 V_2^x = p_1 V_1^x$$

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{V_1}{V_2}\right)^x \quad \therefore p_2 = p_1 \left(\frac{V_1}{V_2}\right)^0 = p_1 (1)$$

$$p_2 = p_1$$

Comprobado la fórmula es correcta

Cálculo de trabajo adiabático Reversible

Es posible calcularlo a partir de $\Delta U = -w$

$$w = -\Delta U$$

Si se conoce T_2 y T_1 , sin embargo se puede obtener

otra forma de cálculo a partir de

$$pV^\gamma = \text{cte}$$

W Adiab R.

$$W = p dv$$

$$p v^{\gamma} = \text{cte}$$

$$W = \frac{\text{cte}}{v^{\gamma}} dv$$

$$p = \frac{\text{cte}}{v^{\gamma}}$$

$$W = \text{cte} \int_{v_1}^{v_2} v^{-\gamma} dv$$

$$W = \text{cte} \frac{V^{-\gamma+1}}{-\gamma+1}$$

v_2
 v_1

$$W = \text{cte} \frac{V^{-\gamma+1}}{1-\gamma}$$

v_2
 v_1

$$W = \frac{p V^\gamma V^{-\gamma+1}}{\gamma-1}$$

v_2
 v_1

$$W = \frac{pV^{\gamma} - p'V'^{\gamma}}{1 - \gamma} \quad \left. \begin{array}{l} V_2 \\ V_1 \end{array} \right\}$$

$$W = \frac{pV}{1 - \gamma} \quad \left. \begin{array}{l} V_2 \\ V_1 \end{array} \right\}$$

$$\Delta pV = nR\Delta T$$

$$W = \frac{p_2V_2 - p_1V_1}{1 - \gamma}$$

$$W_R = \frac{nR(T_2 - T_1)}{1 - \alpha}$$

$$= \frac{(\cancel{\text{mol}})(\cancel{\text{J/molK}})(\cancel{\text{K}})}{1 - \alpha}$$

$$= \text{J}$$

$$W_R = \frac{nR(T_2 - T_1)}{1 - \alpha}$$

$\alpha = \infty$
Isocórico

Analizando la fórmula

$$W = \frac{p_2 V_2 - p_1 V_1}{1 - \gamma} = \frac{(N/m^2)(m^3) - (N/m^2)(m^3)}{1 - \gamma} = \frac{N \cdot m}{1 - \gamma} = J$$

si hay expansión

$$p_2 V_2 < p_1 V_1$$

$$\text{y } W = +$$

$$\gamma > 1 \neq 0$$

si hay compresión

$$p_2 V_2 > p_1 V_1$$

$$\text{y } W = -$$

y no es
negativo

(adimensional)

Exp. Adiab. R.

$$n_1 \rightarrow n_2 = \text{cte}$$

$$p_1 \rightarrow p_2 \quad p_2 < p_1$$

$$T_1 \rightarrow T_2 \quad T_1 > T_2$$

$$V_1 \rightarrow V_2 \quad V_2 > V_1$$

$$q = 0$$

$$\Delta H = - \quad \Delta U = -$$

$$W = + \quad \Delta S = 0$$

Comp. Δ diab R.

$$n_1 \rightarrow n_2 = \text{cte}$$

$$T_1 \rightarrow T_2 \quad T_2 > T_1$$

$$p_1 \rightarrow p_2 \quad p_2 > p_1$$

$$V_1 \rightarrow V_2 \quad V_2 < V_1$$

$$q = 0$$

$$\Delta H = + \quad \Delta U = + \quad W = -$$

$$\Delta U = -W \quad \Delta S = 0$$

Adiabático

$$W_R > W_{IR}$$

$$|\Delta U_R| > |\Delta U_{IR}| \quad \Delta U = -W$$

$$|\Delta T_R| > |\Delta T_{IR}|$$

$$\Delta U = nR \Delta T = nR (T_2 - T_1)$$

Adiabático Exp.

Enfriamiento

$$P_R > P_{IR}$$

$$T_{zR} < T_{zIR}$$

Adiab. IR

$$\text{si } \Delta U = -W$$

$$n\bar{C}_V \Delta T = -p \Delta V$$

$$n\bar{C}_V dT = -p dv$$

$$p = p_2$$

$$n\bar{C}_V (T_2 - T_1) = -p_2 (V_2 - V_1)$$

$$\cancel{n\bar{C}_V} (T_2 - T_1) = -p_2 \left[\cancel{R} \frac{RT_2}{p_2} - \cancel{R} \frac{RT_1}{p_1} \right] \text{ eliminando}$$

$$= p_2 \left[\frac{T_1}{p_1} R - \frac{RT_2}{p_2} \right] \text{ Factorizando}$$

$$V = \frac{nRT}{P}$$

$$\bar{C}_V (T_2 - T_1) = R \left[\frac{P_2 T_1}{P_1} - T_2 \right] \text{ Agrupando}$$

$$T_2 - T_1 = \frac{R}{\bar{C}_V} \left[\frac{P_2 T_1}{P_1} - T_2 \right]$$

$$T_2 = \frac{R}{\bar{C}_V} \left[\frac{P_2 T_1}{P_1} - T_2 \right] + T_1$$

$$T_2 = \frac{\bar{C}_P - \bar{C}_V}{\bar{C}_V} \left[\frac{P_2 T_1}{P_1} - T_2 \right] + T_1$$

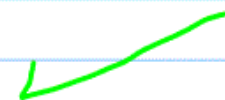
$$T_2 = \gamma^{-1} \left[\frac{P_2 T_1}{P_1} - T_2 \right] + T_1$$

$$T_2 + T_2(\gamma - 1) = \gamma^{-1} \left[\frac{P_2 T_1}{P_1} \right] + T_1$$

$$T_2 + T_2 \gamma - T_2 = T_1 \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right]$$

$$T_2 \gamma = T_1 \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right]$$

$$T_2 = \frac{T_1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right]$$



$$T_2 = \frac{T_1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right]$$

IR

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\gamma - 1/\gamma}$$

R

Comprobando

$$T_2 = \frac{T_1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right] \quad \checkmark$$

en forma general

$$T_2 = \frac{T_1}{X} \left[(X - 1) \frac{P_2}{P_1} + 1 \right] \quad \begin{array}{l} \text{Si fuera isotérmico} \\ X = 1 \end{array}$$

sustituyendo

$$T_2 = \frac{T_1}{1} \left[(1 - 1) \frac{P_2}{P_1} + 1 \right] = T_1 [+1] = T_2 \quad \checkmark$$

Correcto

Obtención IR p vs V

Sustituyendo

$$\frac{P_2 V_2 / \gamma R}{P_1 V_1 / \gamma R} = \frac{1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right]$$

$$T = \frac{P V}{\gamma R}$$

$$P_2 V_2 = \frac{P_1 V_1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right]$$

$$P_2 V_2 = \left[\frac{\cancel{P_1 V_1}}{\gamma} (\gamma - 1) \frac{P_2}{P_1} + \frac{P_1 V_1}{\gamma} \right] \quad \text{eliminando}$$

$$P_2 V_2 = \left[\frac{V_1}{\gamma} (\gamma - 1) P_2 + \frac{P_1 V_1}{\gamma} \right]$$

$$P_2 V_2 \gamma = \left[(\gamma - 1) V_1 P_2 + P_1 V_1 \right]$$

$$P_2 V_2 \gamma = V_1 \left[(\gamma - 1) P_2 + P_1 \right]$$

$$P_2 V_2 = \frac{V_1}{\gamma} \left[(\gamma - 1) P_2 + P_1 \right] \quad \text{dividiendo entre } P_2$$

$$\frac{P_2 V_2}{P_2} = \frac{V_1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_2} + \frac{P_1}{P_2} \right]$$

$$V_2 = \frac{V_1}{\gamma} \left[(\gamma - 1) + \frac{P_1}{P_2} \right] \quad \text{Checar unidades}$$

$$v_2 = \frac{v_1}{\gamma} \left[(\gamma + 1) + \frac{p_1}{p_2} \right] \quad \text{LR}$$

$$p_2 v_2 \gamma = p_1 v_1 \gamma$$

$$v_2 = \frac{p_1 v_1 \gamma}{p_2}$$

$$v_2 = \left(\frac{p_1 v_1 \gamma}{p_2} \right) \frac{1}{\gamma} \quad \text{R}$$

También se puede obtener como función de P_2

$$P_2 V_2 = \frac{V_1}{\gamma} \left[(\gamma - 1) P_2 + P_1 \right]$$

$$P_2 V_2 = \frac{V_1}{\gamma} (\gamma - 1) P_2 + \frac{V_1 P_1}{\gamma}$$

$$P_2 V_2 - \frac{V_1}{\gamma} (\gamma - 1) P_2 = \frac{V_1 P_1}{\gamma}$$

$$P_2 \left[V_2 - \frac{V_1}{\gamma} (\gamma - 1) \right] = \frac{V_1 P_1}{\gamma}$$

$$P_2 = \frac{\frac{V_1 P_1}{\gamma}}{\left[V_2 - \frac{V_1}{\gamma} (\gamma - 1) \right]}$$

Dividido entre $\frac{\gamma}{V_1}$

Cualquier relación de P_2 es válida.

$$P_2 = \frac{\frac{V_1 P_1}{\gamma} \frac{\gamma}{V_1}}{\left[V_2 - \frac{V_1}{\gamma} (\gamma - 1) \right] \frac{\gamma}{V_1}} = \frac{P_1}{\left[\frac{V_2}{V_1} \gamma - (\gamma - 1) \right]}$$

Relación T vs V

Apartir de

$$T_2 = \frac{T_1}{\gamma} \left[(\gamma - 1) \frac{P_2}{P_1} + 1 \right] \quad \text{sustituyendo}$$

$$T_2 = \frac{T_1}{\gamma} \left[(\gamma - 1) \frac{\cancel{nRT_2/v_2}}{\cancel{nRT_1/v_1}} + 1 \right] \quad \text{eliminando}$$

$$\frac{T_2 \gamma}{T_1} = \left[(\gamma - 1) \frac{T_2 / v_2}{T_1 / v_1} + 1 \right]$$

$$\frac{T_2}{T_1} \gamma = \left[(\gamma - 1) \frac{T_2 v_1}{T_1 v_2} + 1 \right]$$

$$\frac{T_2 \gamma}{T_1} = (\gamma - 1) \frac{T_2}{T_1} \frac{v_1}{v_2} + 1$$

$$\frac{T_2}{T_1} \gamma - (\gamma - 1) \frac{T_2}{T_1} \frac{v_1}{v_2} = 1$$

$$\frac{T_2}{T_1} \left[\gamma - (\gamma - 1) \frac{v_1}{v_2} \right] = 1$$

$$T_2 = \frac{T_1}{\left[\gamma - (\gamma - 1) \frac{v_1}{v_2} \right]}$$

Forma general
Inversible

$$T_2 = \frac{T_1}{\left[\gamma - (\gamma - 1) \frac{v_1}{v_2} \right]}$$

Tarea: comparar la Exp. Adiab. Rev.

Contra la Exp. Adiab. Irrev.

Proceso adiabático reversible en gases de comportamiento perfecto e ideal en sistemas cerrados				
Instrucción: Insertar en las celdas de color amarillo los valores correspondientes				
Calculando V_1		proceso	Calculando V_2 adiabático	
p_1 (atm)	5.000	→	γ	1.6667
V_1 (L)	16.400	→	V_2 (L)	14.000
T_1 (K)	1000.000	→	T_2 (K)	1111.251
n_1 (mol)	1.000	→	n_2 (mol)	1.000
Calculando T_1		proceso	Calculando T_2 adiabática	
p_1 (atm)	5.000	→	γ	1.6667
V_1 (L)	16.400	→	V_2 (L)	14.000
T_1 (K)	1000.000	→	T_2 (K)	1111.251
n_1 (mol)	1.000	→	n_2 (mol)	1.000
Calculando p_1		proceso	Calculando p_2 adiabática	
p_1 (atm)	5.000	→	p_2 (atm)	6.50875
V_1 (L)	16.400	→	V_2 (L)	14.000
T_1 (K)	1000.000	→	γ	1.6667
n_1 (mol)	1.000	→	n_2 (mol)	1.000
Calculando n_1		proceso	Calculando T_2 adiabática	
p_1 (atm)	5.000	→	p_2 (atm)	6.50875
V_1 (L)	16.400	→	γ	1.6667
T_1 (K)	1000.000	→	T_2 (K)	1111.250
n_1 (mol)	1.000	→	n_2 (mol)	1.000
R (atmL/molK)	0.0820		C_p (cal/molK)	4.9654
			C_v (cal/molK)	2.9792

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Proceso adiabático Irreversible en gases de comportamiento perfecto e ideal en sistemas cerrados				
Instrucción: Insertar en las celdas de color amarillo los valores correspondientes				
Calculando V_1		proceso	Calculando V_2 adiabático	
p_1 (atm)	5.000	→	γ	1.6667
V_1 (L)	16.400	→	V_2 (L)	14.0005
T_1 (K)	1000.000	→	T_2 (K)	1129.000
n_1 (mol)	1.000	→	n_2 (mol)	1.000
Calculando T_1		proceso	Calculando T_2 adiabática	
p_1 (atm)	5.000	→	γ	1.6667
V_1 (L)	16.400	→	V_2 (L)	14.000
T_1 (K)	1000.000	→	T_2 (K)	1129.037
n_1 (mol)	1.000	→	n_2 (mol)	1.000
Calculando p_1		proceso	Calculando p_2 adiabática	
p_1 (atm)	5.000	→	p_2 (atm)	6.61293
V_1 (L)	16.400	→	V_2 (L)	14.000
T_1 (K)	1000.000	→	γ	1.6667
n_1 (mol)	1.000	→	n_2 (mol)	1.000
Calculando n_1		proceso	Calculando T_2 adiabática	
p_1 (atm)	5.000	→	p_2 (atm)	6.61293
V_1 (L)	16.400	→	γ	1.6667
T_1 (K)	1000.000	→	T_2 (K)	1129.037
n_1 (mol)	1.000	→	n_2 (mol)	1.000
R (atmL/molK)	0.0820		C_p (cal/molK)	4.9654
			C_v (cal/molK)	2.9792

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Adiabático Comp.

$$T_{zR} < T_{zIR}$$

mismo grado de compresión

$$P_{zR} < P_{zIR}$$

Adiab. Comp.

$$\Delta H_R < \Delta H_{IR}$$

$$\Delta U_R < \Delta U_{IR}$$

$$\Delta S_R < \Delta S_{IR}$$

$$w_R < w_{IR}$$

PROCESOS ISOCÓRICOS, ISOBÁRICOS, ADIABÁTICOS e ISOTÉRMICOS EN GASES

Modelo perfecto e ideal Reversibles

Insertar en las celdas de color amarillo los valores correspondientes

Resultados en las celdas de color verde

Constantes de Cp como función de T (cal/molK)

Gas	a	b	c	d	e	m (g)	n (mol)	M (g/mol)
Argón	4.9654e+000					28.0000	1.0000	28.0000

T_1 (K)	T_2 (K)	p_1 (atm)	p_2 (atm)	R (cal/mol K)
1000.00	1121.25	5.0000	6.5087	1.9886

V_1 (L)	V_2 (L)
16.40	14



ΔH (cal)	602.0548
ΔU (cal)	360.9370
ΔS p cte (cal/K)	0.5683
q p cte (cal)	602.0548
w p cte (cal)	241.1178
q isotérmico (cal)	-524.3975

C_p (cal/molK)	4.9654
C_v (cal/molK)	2.9768
ΔS V cte (cal/K)	0.3407
q V cte (cal)	360.9370
w V cte (cal)	0
w isotérmico (cal)	-314.6443

γ	1.6680
w adiabático (cal)	-360.9370
ΔS isotérmico (cal/K)	-0.5244
ΔS adiabático (cal/K)	0
q adiabático (cal)	0
Se cumple la segunda ley de la Termodinámica	

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Con apoyo del programa UNAM-DGAPA-PAPIIME PE-20419

PROCESOS ISOCÓRICOS, ISOBÁRICOS, ADIABÁTICOS e ISOTÉRMICOS EN GASES

Modelo perfecto e ideal Irreversibles

Insertar en las celdas de color amarillo los valores correspondientes

Resultados en las celdas de color verde

Constantes de Cp como función de T (cal/molK)

Gas	a	b	c	d	e	m (g)	n (mol)	M (g/mol)
Argón	4.9654e+000					28.0000	1.0000	28.0000

T_1 (K)	T_2 (K)	p_1 (atm)	p_2 (atm)	R (cal/mol K)
1000.00	1129.00	5.0000	6.6129	1.9886
V_1 (L)	V_2 (L)			
16.40	14			



ΔH (cal)	640.5366
ΔU (cal)	384.0072
ΔS p cte (cal/K)	0.6025
q p cte (cal)	640.5366
w p cte (cal)	256.5294
q isotérmico (cal)	-384.1675

C_p (cal/molK)	4.9654
C_v (cal/molK)	2.9768
ΔS V cte (cal/K)	0.3612
q V cte (cal)	384.0072
w V cte (cal)	0
w isotérmico (cal)	-384.1675

γ	1.6680
w adiabático (cal)	-384.0072
ΔS isotérmico (cal/K)	-0.3842
ΔS adiabático (cal/K)	0.04654
q adiabático (cal)	0
Se cumple la segunda ley de la termodinámica	

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Con apoyo del programa UNAM-DGAPA-PAPIME PE-20419

$$\Delta U = q - w$$

$$p = \frac{nRT}{V}$$

$$q = \Delta U + w$$

$$q = n \bar{C}_v dT + p dv$$

$$q = n \bar{C}_v dT + nRT \frac{dv}{V}$$

$$\frac{\delta q}{T} = ds$$

$$\int_1^2 ds = \int_{T_1}^{T_2} \frac{n \bar{C}_V dT}{T} + \frac{nRT}{T} \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\Delta S = n \bar{C}_V \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1}$$

